Apache SystemML: Declarative Large-Scale Machine Learning

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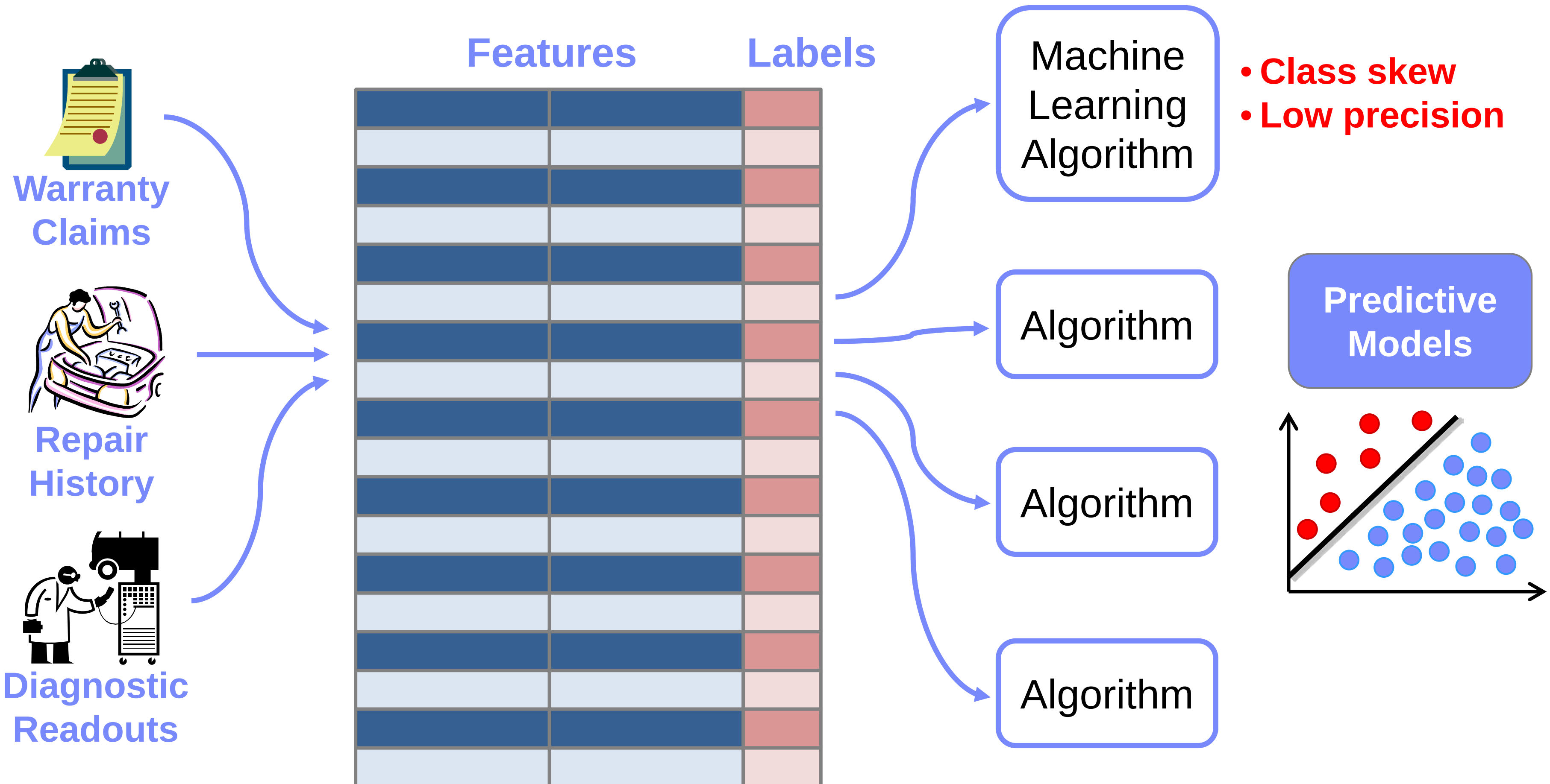
Case Study: An Automobile Manufacturer

- **Goal:** Design a model to predict car reacquisition



Result: 25x improvement in precision!

Common Patterns Across Customers

- **Algorithm customization**
- **Changes in feature set**
- **Changes in data size**
- **Quick iteration**

→

Custom Analytics

Declarative Machine Learning

Abstraction: The Good, the Bad and the Ugly

[adapted from Peter Alvaro: "I See What You Mean“, **Strange Loop, 2015**]

Platform Independence

Data Independence **Adaptivity**

Simple & Analysis-Centric **Efficiency & Performance**

```
q = t(X) %*% (w * (X %*% v))
```

The Ugly: Expectations ≠ Reality

(Missing) Rewrites

(Missing) Size Information

Complex Control Flow

Operator Selection

(Implicit) Copy-on-Write

Distributed Operations

Data Skew

Latency

Local / Remote Memory Budgets

Load Imbalance

Distributed Storage

➔ Understanding of optimizer and runtime techniques underpinning declarative, large-scale ML

Tutorial Outline

- **Case Study and Motivation (Flash)** 5min
- **SystemML Overview, APIs, and Tools** 30min
- **Common Framework** 15min
- **SystemML's Optimizer (w/ Hands-On-Labs)** 45min

High-Level SystemML Architecture

DML (**D**eclarative **M**achine **L**earning **L**anguage)

DML Scripts

↓

Language

Compiler

Runtime

This Talk (Compiler, Runtime)

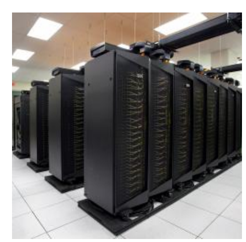
In-Memory Single Node
(scale-up)

Java

since 2012

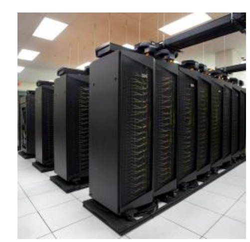
Hadoop or Spark Cluster
(scale-out)

hadoop



since 2010/11

APACHE Spark



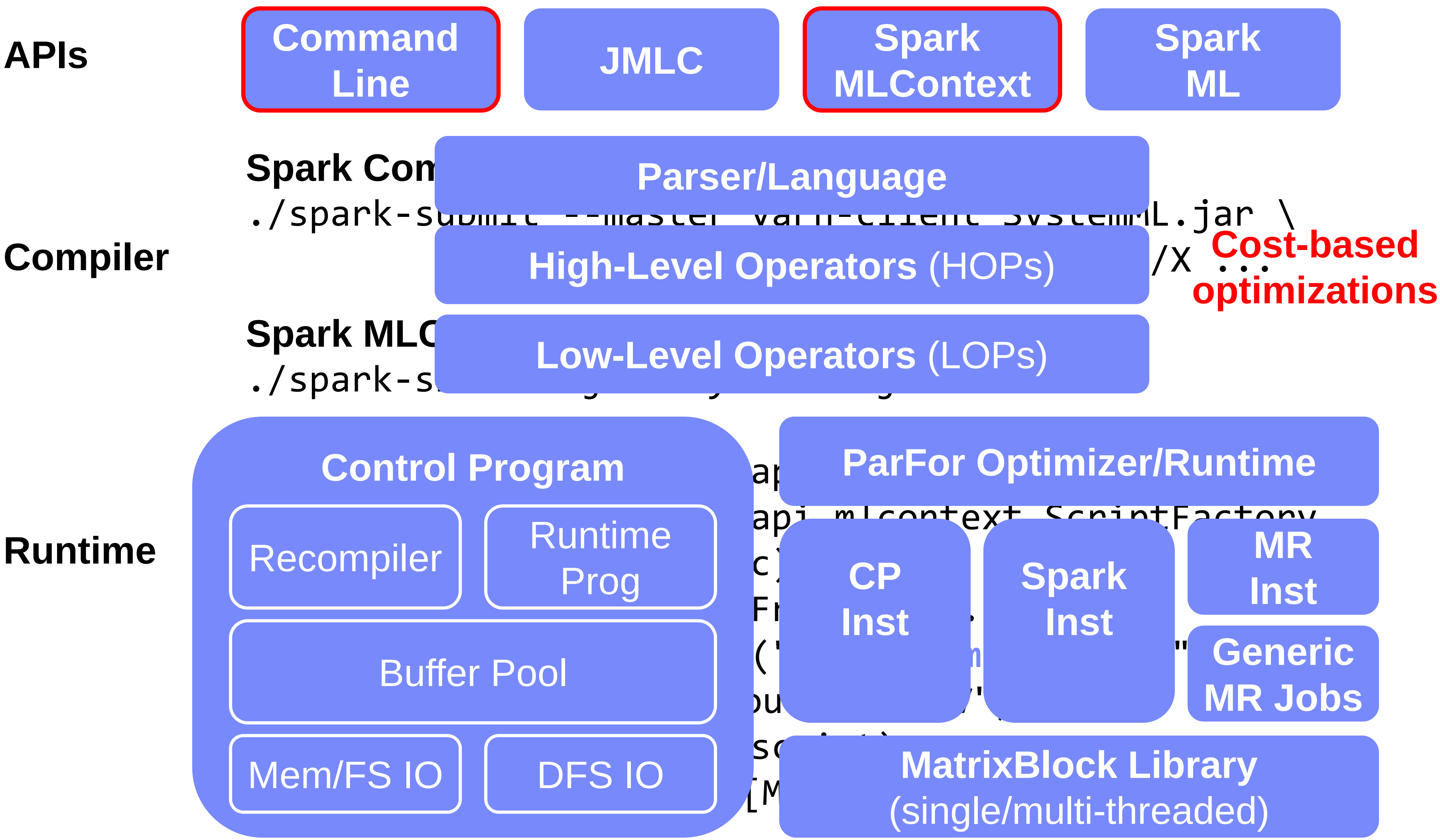
since 2015

Running Example

- **Collaborative filtering**
  - Matrix completion
  - Low rank factorization
    $\mathbf{X \approx U\,V^T}$
- **ALS-CG** (alternating least squares via conjugate gradient)
  - L2-regularized squared loss
  - Repeatedly fixes one factor and optimizes the other factor
  - Conjugate Gradient to solve least-squares problems jointly

```
X = read($inFile);
r = $rank; lambda = $lambda; mi = $maxiter;
U = rand(rows=nrow(X), cols=r, min=-1.0, max=1.0);
V = rand(rows=r, cols=ncol(X), min=-1.0, max=1.0);
W = (X != 0); mii = r; i = 0; is_U = TRUE;
while( i < mi ) {
   i = i + 1; ii = 1;
   if( is_U )
      G = (W *(U %*% V - X)) %*% t(V) + lambda * U;
   else ...
   norm_G2 = sum(G^2); norm_R2 = norm_G2; ...
   while( norm_R2 > 10E-9 * norm_G2 & ii <= mii ) {
      if( is_U ) {
         HS = (W * (S %*% V)) %*% t(V) + lambda * S;
         alpha = norm_R2 / sum(S * HS);
         U = U + alpha * S;
      } else {...}
      ...
   }
   is_U = !is_U;
}
write(U, $outUFile, format="text");
write(V, $outVFile, format="text");
```

SystemML Architecture and APIs



Basic Setup and Hands-On-Lab

- **Downloads** (https://systemml.apache.org/download.html)
  - **systemml-0.10.0-incubating** (default cluster setup)
  - **systemml-0.10.0-incubating-standalone** (self-contained local setup)

- **Example script**
  - Test.dml: `print(sum(rand(rows=1000,cols=1000)));`

- **Basic invocation** (with various execution types)
  - Hadoop (hybrid)

    ```
    hadoop jar SystemML.jar -f Test.dml …
    ```

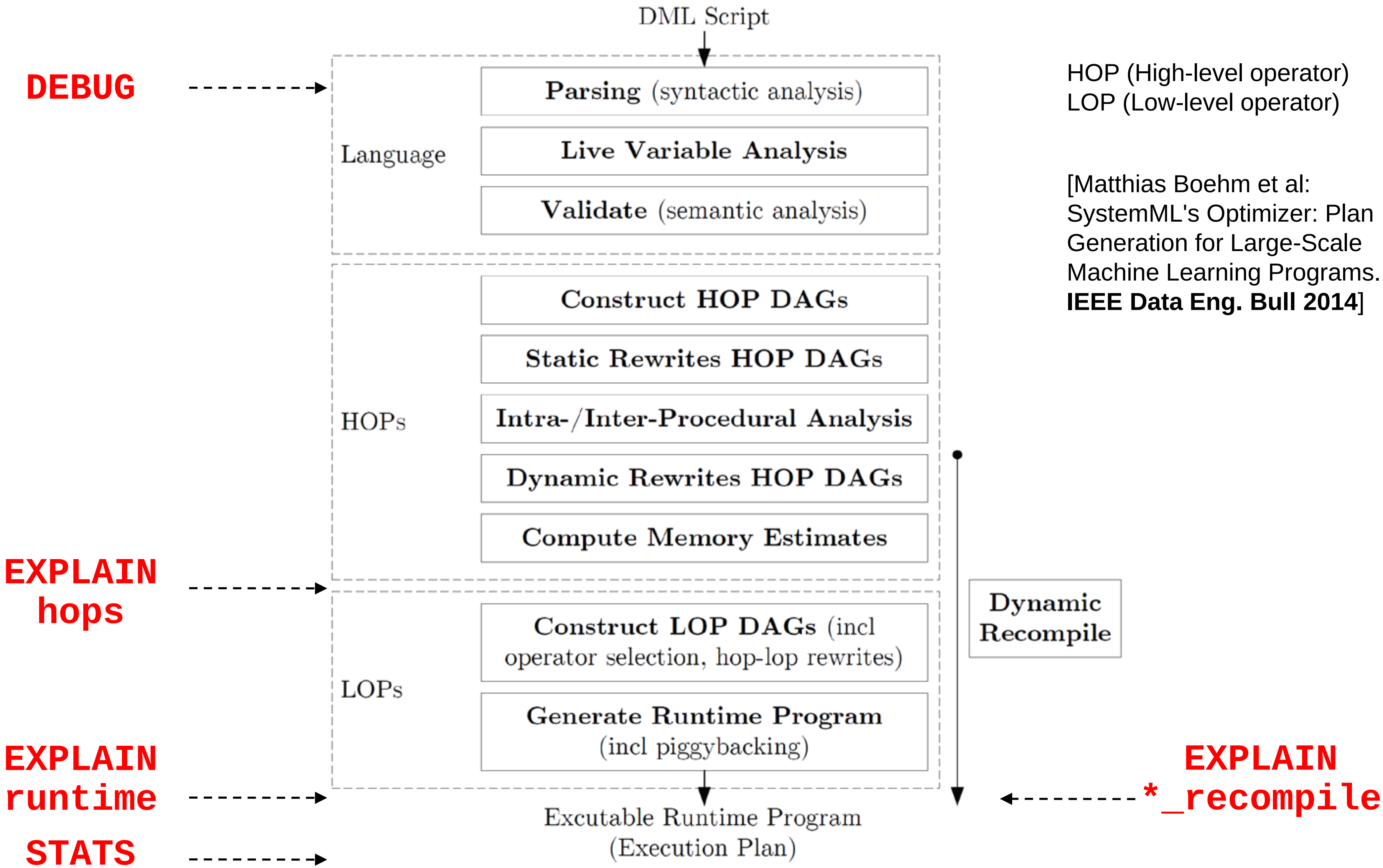
  - Spark (hybrid_spark)

    ```
    ./spark_submit –master yarn-client SystemML.jar -f Test.dml …
    ```

  - **Standalone (singlenode, …)**

    ```
    ./runStandaloneSystemML.sh Test.dml …
    java -cp … -f Test.dml …
    ```

SystemML's Compilation Chain / Overview Tools



HOP (High-level operator)
LOP (Low-level operator)

[Matthias Boehm et al: SystemML's Optimizer: Plan Generation for Large-Scale Machine Learning Programs. **IEEE Data Eng. Bull 2014**]

Explain (Understanding Execution Plans)

- **Overview**
  - Shows generated execution plan
  - Introduced 05/2014 for internal usage
  - ➔ **Important tool for understanding/debugging optimizer choices!**

- **Usage**

```
hadoop jar SystemML.jar -f test.dml -explain
     [hops | runtime | hops_recompile | runtime_recompile]
```

  - **Hops:** Program with hop dags after optimization
  - **Runtime** (default): Program with runtime instructions
  - **Hops_recompile**: Hops + hop dag after every recompilation
  - **Runtime_recompile**: Runtime instructions after every recompilation

Explain: Understanding HOP DAGs

- **Example DML script** (simplified LinregDS)

```
X = read($1);
y = read($2);
intercept = $3;
lambda = $4;
...
if( intercept == 1 ) {
   ones = matrix(1,nrow(X),1);
   X = append(X, ones);
}
I = matrix(1, ncol(X), 1);
A = t(X) %*% X + diag(I)*lambda;
b = t(X) %*% y;
beta = solve(A, b);
...
write(beta, $5);
```

Invocation:

```
hadoop jar SystemML.jar
-f LinregDS.dml
-args mboehm/X mboehm/y
0 0 mboehm/beta
```

Scenario:

X: 100,000 x 1,000, 1.0
y: 100,000 x 1, 1.0
(800MB, 200+GFlop)

Explain: Understanding HOP DAGs (2)

- **Explain Hops**

```
16/09/08 09:43:13 INFO api.DMLScript: EXPLAIN (HOPS):
# Memory Budget local/remote = 56627MB/1434MB/1434MB
# Degree of Parallelism (vcores) local/remote = 24/96/48
PROGRAM
--MAIN PROGRAM
----GENERIC (lines 1-4) [recompile=false]
------(10) PRead X [100000,1000,1000,1000,100000000] [0,0,763 -> 763MB], CP
------(11) TWrite X (10) [100000,1000,1000,1000,100000000] [763,0,0 -> 763MB], CP
------(21) PRead y [100000,1,1000,1000,100000] [0,0,1 -> 1MB], CP
------(22) TWrite y (21) [100000,1,1000,1000,100000] [1,0,0 -> 1MB], CP
------(24) TWrite intercept [0,0,-1,-1,-1] [0,0,0 -> 0MB], CP
------(26) TWrite lambda [0,0,-1,-1,-1] [0,0,0 -> 0MB], CP
----GENERIC (lines 11-16) [recompile=false]
------(42) TRead X [100000,1000,1000,1000,100000000] [0,0,763 -> 763MB], CP
------(54) r(t) (42) [1000,100000,1000,1000,100000000] [763,0,763 -> 1526MB]
------(55) ba(+*) (54,42) [1000,1000,1000,1000,-1] [1526,8,8 -> 778MB], CP
------(43) TRead y [100000,1,1000,1000,100000] [0,0,1 -> 1MB], CP
------(61) ba(+*) (54,43) [1000,1,1000,1000,-1] [764,0,0 -> 764MB], CP
------(62) b(solve) (55,61) [1000,1,1000,1000,-1] [8,8,0 -> 15MB], CP
------(68) PWrite beta (62) [1000,1,-1,-1,-1] [0,0,0 -> 0MB], CP
```

Cluster Characteristics

Program Structure (incl recompile)

Unrolled HOP DAG

Explain: Understanding HOP DAGs (3)

- **Explain Hops (cont')**

```
----GENERIC (lines 11-16) [recompile=false]
------(42) TRead X [100000,1000,1000,1000,100000000] [0,0,763 -> 763MB], CP
------(54) r(t) (42) [1000,100000,1000,1000,100000000] [763,0,763 -> 1526MB]
------(55) ba(+*) (54,42) [1000,1000,1000,1000,-1] [1526,8,8 -> 778MB], CP
```

  – HOP ID
  – HOP opcode
  – HOP input data dependencies (via HOP IDs)
  – HOP output matrix characteristics (rlen, clen, brlen, bclen, nnz)
  – Hop memory estimates (inputs, intermediates, output → operation mem)
  – Hop execution type (CP/SP/MR)
  – Optional: indicators of rblk, chkpt, repart, in-place, etc

  – Notes
    - Not all worst-case estimates for dims/memory visible
    - Hops without execution type don't have corresponding lops (e.g., r(t))

Explain: Understanding Runtime Plans (1)

- **Explain Runtime** (simplified filenames, removed rmvar)

Literally a string representation of runtime instructions

```
16/09/08 09:44:22 INFO api.DMLScript: EXPLAIN (RUNTIME):
# Memory Budget local/remote = 56627MB/1434MB/1434MB
# Degree of Parallelism (vcores) local/remote = 24/96/48
PROGRAM ( size CP/MR = 0/0 )
--MAIN PROGRAM
----GENERIC (lines 1-4) [recompile=false]
------CP createvar pREADX mboehm/X false MATRIX binaryblock 100000 1000 1000 1000 100000000
------CP createvar pREADy mboehm/y false MATRIX binaryblock 100000 1 1000 1000 100000
------CP assignvar 0.SCALAR.INT.true intercept.SCALAR.INT
------CP assignvar 0.SCALAR.INT.true lambda.SCALAR.INT
------CP cpvar pREADX X
------CP cpvar pREADy y
----GENERIC (lines 11-16) [recompile=false]
------CP createvar _mVar2 …/_t0/temp1 true MATRIX binaryblock 1000 1000 1000 1000 -1
------CP tsmm X.MATRIX.DOUBLE _mVar2.MATRIX.DOUBLE LEFT 24
------CP createvar _mVar3 …/_t0/temp2 true MATRIX binaryblock 1 100000 1000 1000 100000 copy
------CP r' y.MATRIX.DOUBLE _mVar3.MATRIX.DOUBLE 24
------CP createvar _mVar4 …/_t0/temp3 true MATRIX binaryblock 1 1000 1000 1000 -1 copy
------CP ba+* _mVar3.MATRIX.DOUBLE X.MATRIX.DOUBLE _mVar4.MATRIX.DOUBLE 24
------CP createvar _mVar5 …/_t0/temp4 true MATRIX binaryblock 1000 1 1000 1000 -1 copy
------CP r' _mVar4.MATRIX.DOUBLE _mVar5.MATRIX.DOUBLE 24
------CP createvar _mVar6 …/_t0/temp5 true MATRIX binaryblock 1000 1 1000 1000 -1 copy
------CP solve _mVar2.MATRIX.DOUBLE _mVar5.MATRIX.DOUBLE _mVar6.MATRIX.DOUBLE
------CP write _mVar6.MATRIX.DOUBLE mboehm/beta.SCALAR.STRING.true textcell
```

Stats (Profiling Runtime Statistics)

- **Overview**
  - Profiles and shows aggregated runtime statistics
  - Introduced 01/2014 for internal usage
  - **Important tool for understanding runtime characteristics and profiling**

- **Usage**

```
hadoop jar SystemML.jar -f test.dml -stats
```

Stats: Understanding Runtime Statistics

- **Statistics**

```
16/09/08 09:47:21 INFO api.DMLScript: SystemML Statistics:
Total execution time:           4.518 sec.
Number of compiled MR Jobs:     0.
Number of executed MR Jobs:     0.
Cache hits (Mem, WB, FS, HDFS): 5/0/0/2.
Cache writes (WB, FS, HDFS):    5/0/1.
Cache times (ACQr/m, RLS, EXP): 0.830/0.000/0.002/0.204 sec.
HOP DAGs recompiled (PRED, SB): 0/0.
HOP DAGs recompile time:        0.000 sec.
Total JIT compile time:         0.978 sec.
Total JVM GC count:             2.
Total JVM GC time:              0.184 sec.
Heavy hitter instructions (name, time, count):
-- 1)   tsmm    3.602 sec       1
-- 2)   solve   0.585 sec       1
-- 3)   write   0.205 sec       1
-- 4)   ba+*    0.070 sec       1
-- 5)   r'      0.035 sec       2
-- 6)   createvar       0.000 sec       7
-- 7)   rmvar   0.000 sec       8
-- 8)   cpvar   0.000 sec       2
-- 9)   assignvar       0.000 sec       2
```

Total exec time

Buffer pool stats

Dynamic recompilation stats

JVM stats (JIT, GC)

Heavy hitter instructions (incl. buffer pool times)

optional: parfor and update in-place stats (if applicable)

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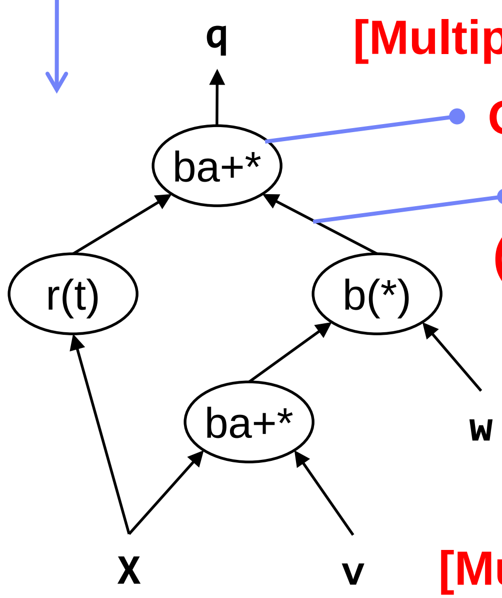
ML Program Compilation

- **Script**

```
while(...) {
    q = t(X) %*% (w * (X %*% v))
    ...
}
```

- **Operator DAG**
  - a.k.a. "graph"
  - a.k.a. intermediate representation (IR)
- **Runtime plans**
  - Interpreted plans
  - Compiled runtime plans (e.g., instructions)



[Multiple] DAG roots (outputs)

Operation

Data dependency
(incl data flow properties)

[Multiple] Consumers of intermediates

No cycles

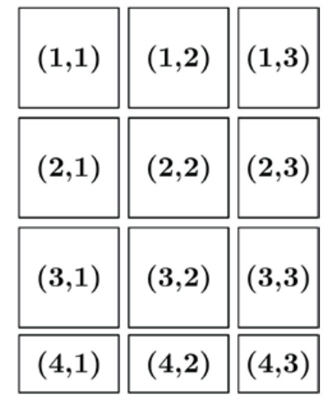
[Multiple] DAG leafs (inputs)

```
SPARK mapmmchain X.MATRIX.DOUBLE w.MATRIX.DOUBLE v.MATRIX.DOUBLE
_mVar4.MATRIX.DOUBLE XtwXv
```

Distributed Matrix Representation

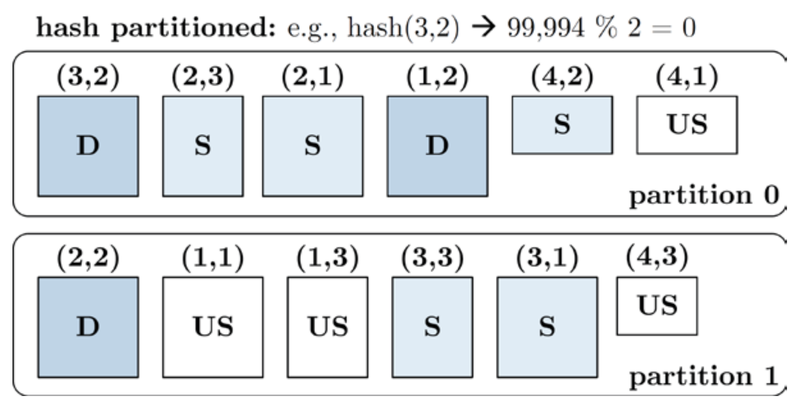
- **Collection of “matrix blocks” (and keys)**
  – a.k.a. “tiles”, a.k.a. “chunks”
  – Bag semantics (duplicates, unordered)
  – Logical (fixed-size) blocking
    + join processing / independence
    - (sparsity skew)
  – E.g., SystemML on Spark:
    `JavaPairRDD<MatrixIndexes,MatrixBlock>`
  – Blocks encoded independently (dense/sparse)

Logical blocking
3,400x2,700 matrix
(w/ B_c=1,000)



- **Partitioning**
  – Logical partitioning
    (e.g., row-/column-wise)
  – Physical partitioning
    (e.g., Hash / Grid)

Physical blocking and partitioning



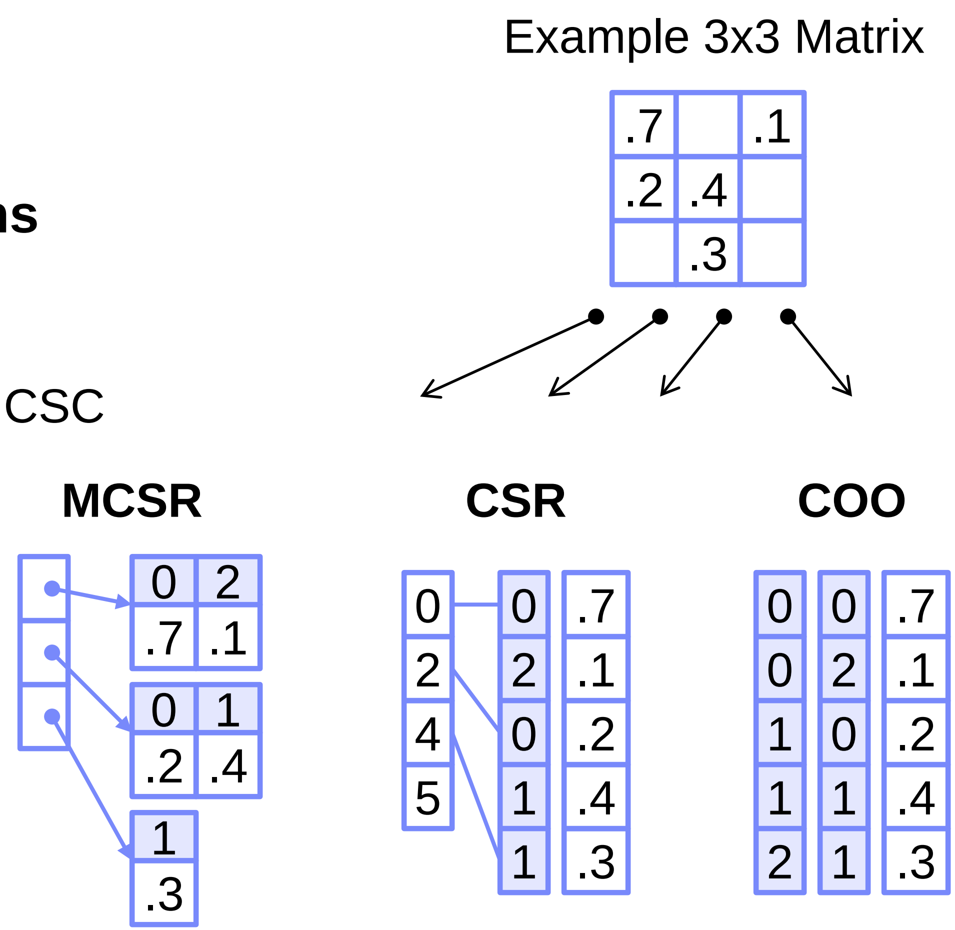
Distributed Matrix Representation (2)

- **Matrix block**
  - Most operations defined here
  - Local matrix: single block
  - Different representations

- **Common block representations**
  - Dense (linearized arrays)
  - MCSR (modified CSR)
  - CSR (compressed sparse rows), CSC
  - COO (Coordinate matrix)
  - …

Dense (row-major)

.7	0	.1	.2	.4	0	0	.3	0



Common Workload Characteristics

- **Common operations**
  - Matrix-Vector **X v** (e.g., LinregCG, Logreg, GLM, L2SVM, PCA)
  - Vector-Matrix $\mathbf{v}^T \mathbf{X}$ (e.g., LinregCG, LinregDS, Logreg, GLM, L2SVM)
  - MMChain $\mathbf{X}^T(\mathbf{w} * \mathbf{X}\ \mathbf{v})$ (e.g., LinregCG, Logreg, GLM)
  - TSMM $\mathbf{X}^T\mathbf{X}$ (e.g., LinregDS, PCA)

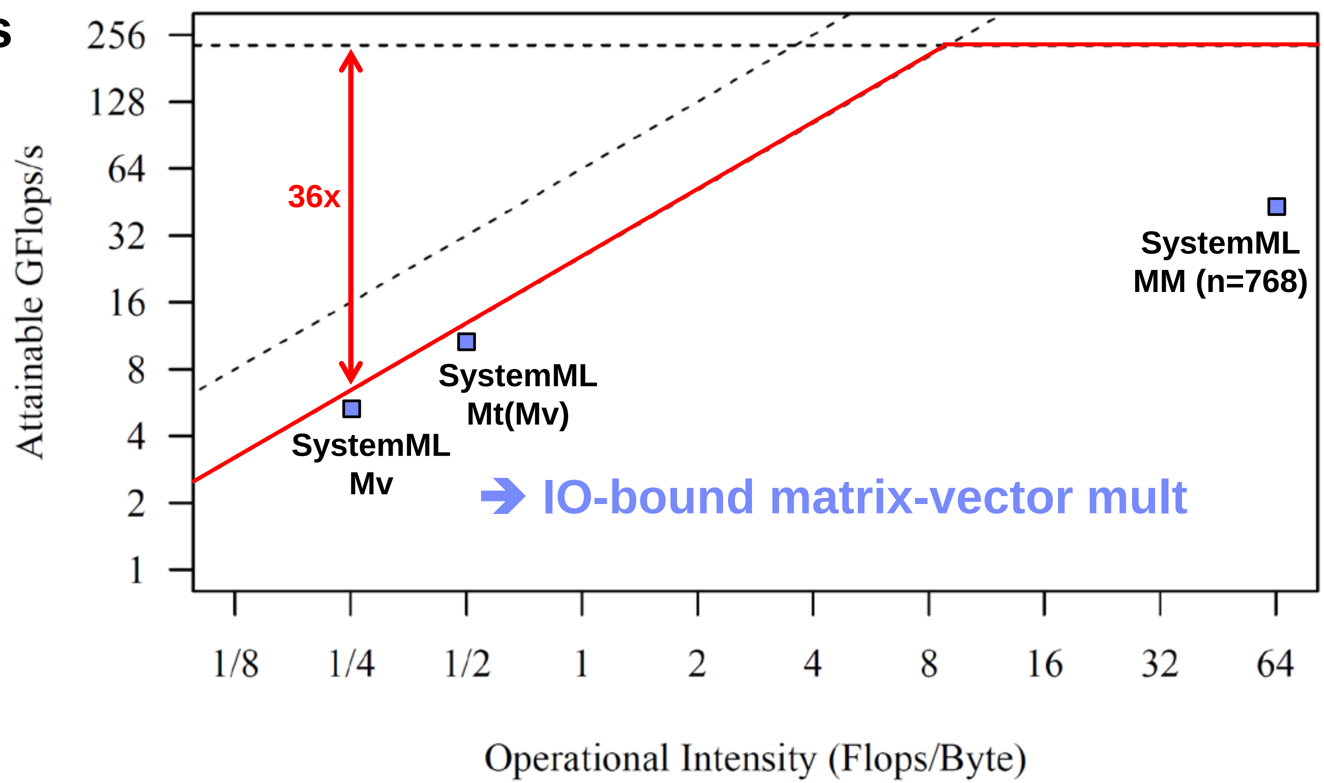
- **Common data characteristics**
  - Tall and skinny matrices
  - Wide matrices often sparse
  - Non-uniform sparsity
  - Transformed data often w/ low column cardinality
  - Column correlations

LinregCG (Conjugate Gradient)

```
X = read($1); # n x m matrix
y = read($2); # n x 1 vector
maxi = 50; lambda = 0.001;
intercept = $3;
...
r = -(t(X) %*% y);
norm_r2 = sum(r * r); p = -r;
w = matrix(0, ncol(X), 1); i = 0;
while(i<maxi & norm_r2>norm_r2_trgt) {
   q = (t(X) %*% (X %*% p))+lambda*p;
   alpha = norm_r2 / sum(p * q);
   w = w + alpha * p;
   old_norm_r2 = norm_r2;
   r = r + alpha * q;
   norm_r2 = sum(r * r);
   beta = norm_r2 / old_norm_r2;
   p = -r + beta * p; i = i + 1;
}
write(w, $4, format="text");
```

Excursus: Roofline Analysis Matrix-Vector Multiply

- **Single Node:** 2x6 E5-2440 @2.4GHz–2.9GHz, DDR3 RAM @1.3GHz (ECC)
  - Max mem bandwidth (local): 2 sock x 3 chan x 8B x 1.3G trans/s → **2 x 32GB/s**
  - Max mem bandwidth (QPI, full duplex) → **2 x 12.8GB/s**
  - Max floating point ops: 12 cores x 2*4dFP-units x 2.4GHz → **2 x 115.2GFlops/s**
- **Roofline Analysis**
  - Processor performance
  - Off-chip memory traffic



[S. Williams, A. Waterman, D. A. Patterson: Roofline: An Insightful Visual Performance Model for Multicore Architectures. **Commun. ACM** 52(4): 65-76 (2009)]

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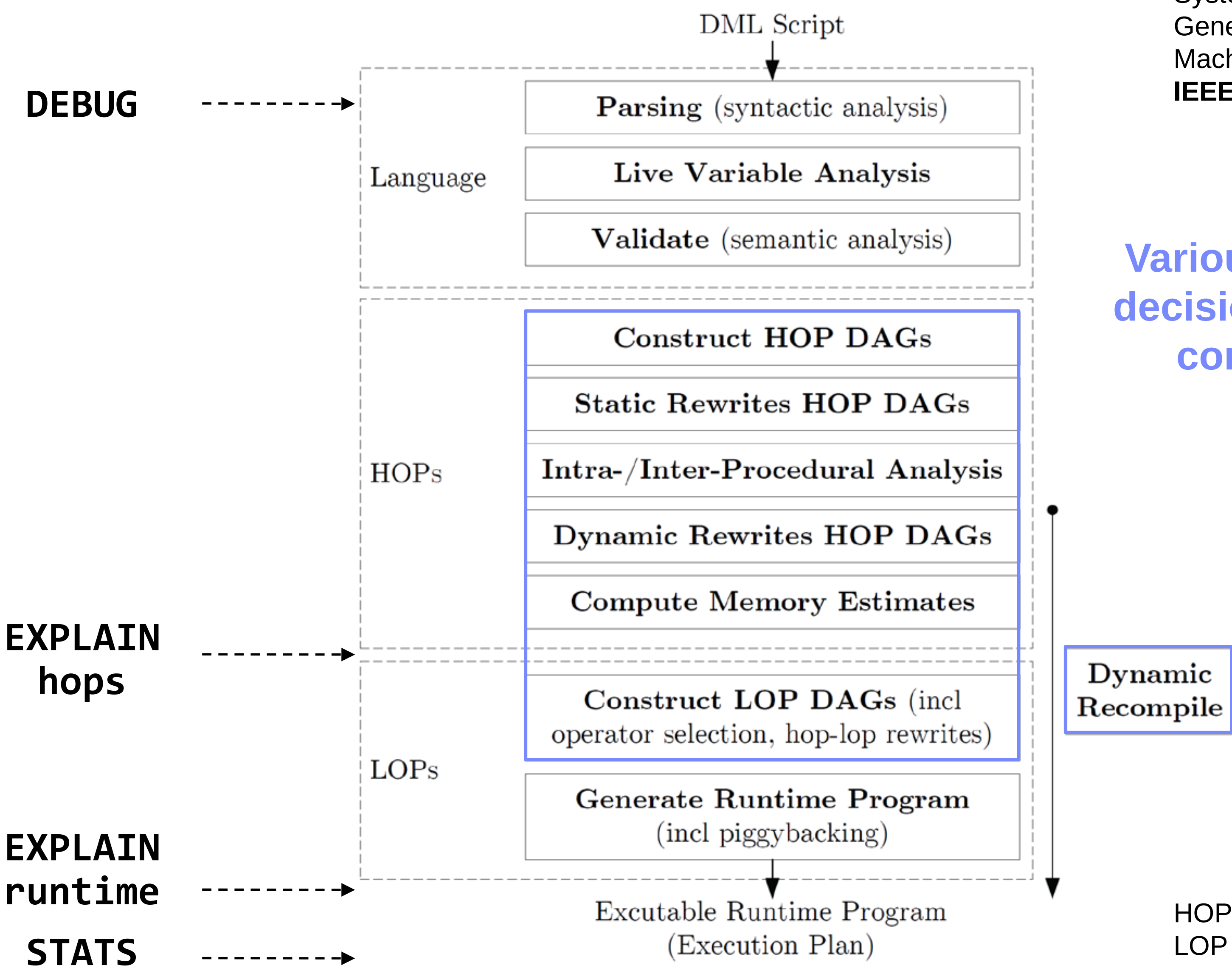
Recap: SystemML's Compilation Chain

[Matthias Boehm et al: SystemML's Optimizer: Plan Generation for Large-Scale Machine Learning Programs. **IEEE Data Eng. Bull 2014**]



Various optimization decisions at different compilation steps

- IPA
- ParFor Opt
- Resource Opt
- GDF Opt

HOP (High-level operator)
LOP (Low-level operator)

Basic HOP and LOP DAG Compilation

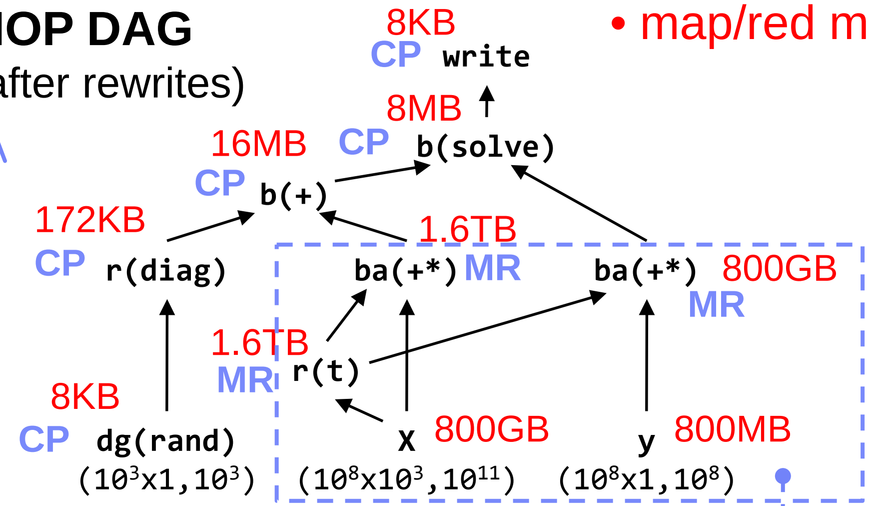
Cluster Config:
- client mem: 4 GB
- map/red mem: 2 GB

Example LinregDS

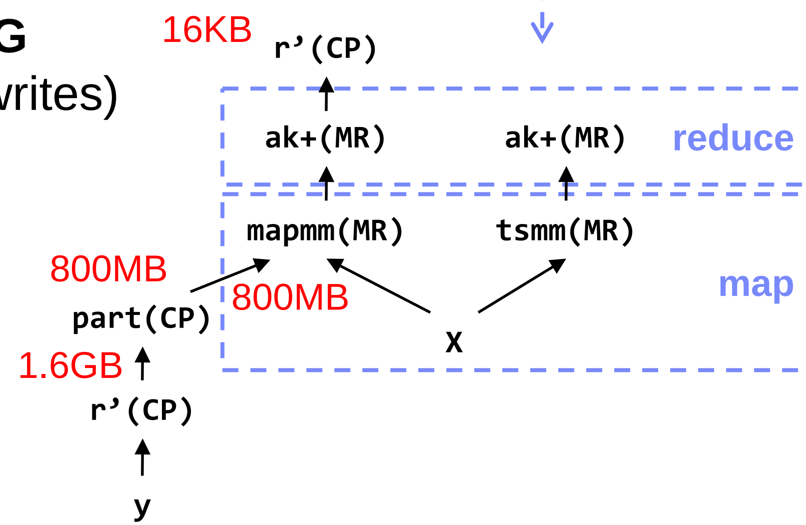
```
X = read($1);
y = read($2);
intercept = $3;
lambda = 0.001;
...
if( intercept == 1 ) {
  ones = matrix(1,nrow(X),1);
  X = append(X, ones);
}
I = matrix(1, ncol(X), 1);
A = t(X) %*% X + diag(I)*lambda;
b = t(X) %*% y;
beta = solve(A, b);
...
write(beta, $4);
```

Scenario:
X: $10^8 \times 10^3$, 10^{11}
y: $10^8 \times 1$, 10^8

HOP DAG
(after rewrites)



LOP DAG
(after rewrites)



➔ **Hybrid Runtime Plans:**
- **Size propagation over ML programs**
- **Worst-case sparsity / memory estimates**
- **Integrated CP / MR / Spark runtime**

Static and Dynamic Rewrites

- **Types of Rewrites**
  - Static: size-independent rewrites
  - Dynamic: size-dependent rewrites
- **Examples Static Rewrites**
  - Common Subexpression Elimination
  - Constant Folding
  - Static Algebraic Simplification Rewrites
  - Branch Removal
  - Right/Left Indexing Vectorization
  - For Loop Vectorization
  - Checkpoint injection (caching)
  - Repartition injection
- **Examples Dynamic Rewrites**
  - Matrix Multiplication Chain Optimization
  - Dynamic Algebraic Simplification Rewrites

Cascading rewrite effect (enables other rewrites, IPA, operator selection)

High performance impact (direct/indirect)

Example Static Simplification Rewrites

- **Static Simplification Rewrites** (size-independent patterns)

Rewrite Category	Static Patterns

Remove Unnecessary Operations	`t(t(X)), X/1, X*1, X-0, -(-X)` → `X`
	`matrix(1,)/X` → `1/X` **`sum(t(X))` → `sum(X)`**
	`rand(,min=-1,max=1)*7` → `rand(,min=-7,max=7)`
	`-rand(,min=-2,max=1)` → `rand(,min=-1,max=2)`
	`t(cbind(t(X),t(Y)))` → `rbind(X,Y)`

Simplify Bushy Binary	`(X*(Y*(Z%*%v)))` → `(X*Y)*(Z%*%v)`

Binary to Unary	`X+X` → `2*X` **`X*X` → `X^2`** `X-X*Y` → `X*(1-Y)`
	`X*(1-X)` → `sprop(X)` `1/(1+exp(-X))` → `sigmoid(X)`
	`X*(X>0)` → `selp(X)` **`(X-7)*(X!=0)` → `X -nz 7`**
	`(X!=0)*log(X)` → `log_nz(X)`
	`aggregate(X,y,count)` → `aggregate(y,y,count)`

Simplify Permutation Matrix Construction	`outer(v,seq(1,N),"==")` → `rexpand(v,max=N,row)` `table(seq(1,nrow(v)),v,N)` → `rexpand(v,max=N,row)`
Simplify Operation over Matrix Multiplication	**`trace(X%*%Y)` → `sum(X*t(Y))`** `(X%*%Y)[7,3]` → `X[7,] %*% Y[,3]`

Example Dynamic Simplification Rewrites

- **Dynamic Simplification Rewrites** (size-dependent patterns)

<table>
<tr><th>Rewrite Category</th><th>Dynamic Patterns</th></tr>
<tr><td>Remove / Simplify Unnecessary Indexing</td><td>X[a:b,c:d] = Y → X = Y iff dims(X)=dims(Y)
X = Y[, 1] → X = Y iff dims(X)=dims(Y)
X[,1]=Y;X[,2]=Z → X=cbind(Y,Z) iff ncol(X)=2,col</td></tr>
<tr><td>Fuse / Pushdown Operations</td><td>t(rand(10, 1)) → rand(1, 10) iff nrow/ncol=1
sum(diag(X)) → trace(X) iff ncol(X)>1
diag(X)*7 → diag(X*7) iff ncol(X)=1
sum(X^2) →t(X)%*%X, →sumSq(X) iff ncol(X)=1, >1</td></tr>
<tr><td>Remove Empty / Unnecessary Operations</td><td>X%*%Y → matrix(0,…) iff nnz(X)=0|nnz(Y)=0
X*Y → matrix(0,…), X+Y→X, X-Y→X iff nnz(Y)=0
round(X)→matrix(0), t(X)→matrix(0) iff nnz(X)=0
X*(Y%*%matrix(1,)) → X*Y iff ncol(Y)=1</td></tr>
<tr><td>Simplify Aggregates / Scalar Operations</td><td>rowSums(X) →sum(X) →X iff nrow(X)=1, ncol(X)=1
rowSums(X*Y) → X%*%t(Y) iff nrow(Y)=1
X*Y → X*as.scalar(Y) iff nrow(Y)=1&ncol(Y)=1</td></tr>
<tr><td>Simplify Diag Matrix Multiplications</td><td>diag(X)%*%Y → Y*X iff ncol(X)=1&ncol(Y)>1
diag(X%*%Y)->rowSums(X*t(Y)) iff ncol(Y)>1</td></tr>
</table>

Hands-On Labs: Rewrites and Handling of Size Information

- **Exercise 1: Sum-Product Rewrite:** `sum(A %*% t(B))`
  - a) What's happening for A:=[900x1000], B:=[700,1000]
  - b) What's happening for A:=[900x1], B:=[700x1]

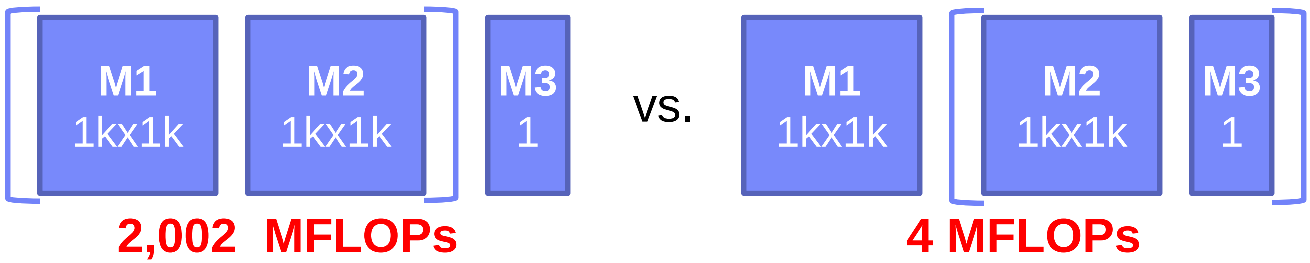
- **Exercise 2: Matrix Multiplication Chains:** `A %*% B %*% C %*% D %*% E`
  - What's happening as we change dimensions of A, B, C, D, E (start with dimensions given on slide 17)

- **Exercise 3: Dynamic Recompilation**
  - What's happening during compilation/runtime to gather size information

```
if( $1 == 1 ) {
    Y = rand(rows=nrow(X), cols=1, min=1, max=maxval);
    X = cbind(X, table(seq(1,nrow(Y)),Y));}
print(sum(X));
```

Matrix Multiplication Chain Optimization

- **Problem**
  - Given a matrix multiplication chain (sequence) of n matrices $M_1, M_2, \ldots M_n$
  - Matrix multiplication is associative
  - Find the optimal full parenthesization of the product $M_1 M_2 \ldots M_n$

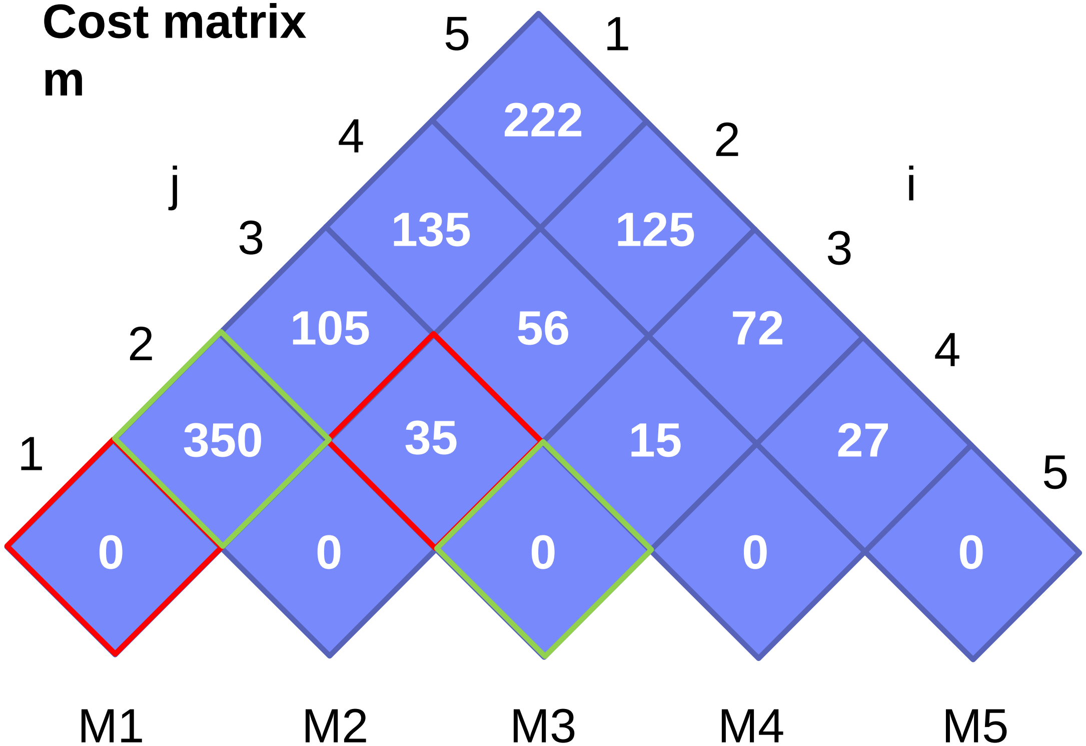


- **Search Space Characteristics**
  - Naïve exhaustive search: Catalan numbers → $\Omega(4^n / n^{3/2})$
  - Few distinct subproblems: any i and j, w/ $1 \leq i \leq j \leq n$: $\Theta(n^2)$
  - DP characteristics apply: (1) optimal substructure, (2) overlapping subproblems
  - Text book dynamic programming algorithm: $\Theta(n^3)$ time, $\Theta(n^2)$ space [T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: Introduction to Algorithms, Third Edition, **The MIT Press**, pages 370-377, 2009]
  - Best known algorithm: $O(n \log n)$ [T. C. Hu, M. T. Shing: Computation of Matrix Chain Products. Part II. **SIAM J. Comput.** 13(2): 228-251, 1984]

Matrix Multiplication Chain Optimization (2)

M1	M2	M3	M4	M5
10x7	7x5	5x1	1x3	3x9

Cost matrix m



m[1,3] = min(
m[1,1] + m[2,3] + p1p2p4,
m[1,2] + m[3,3] + p1p3p4)

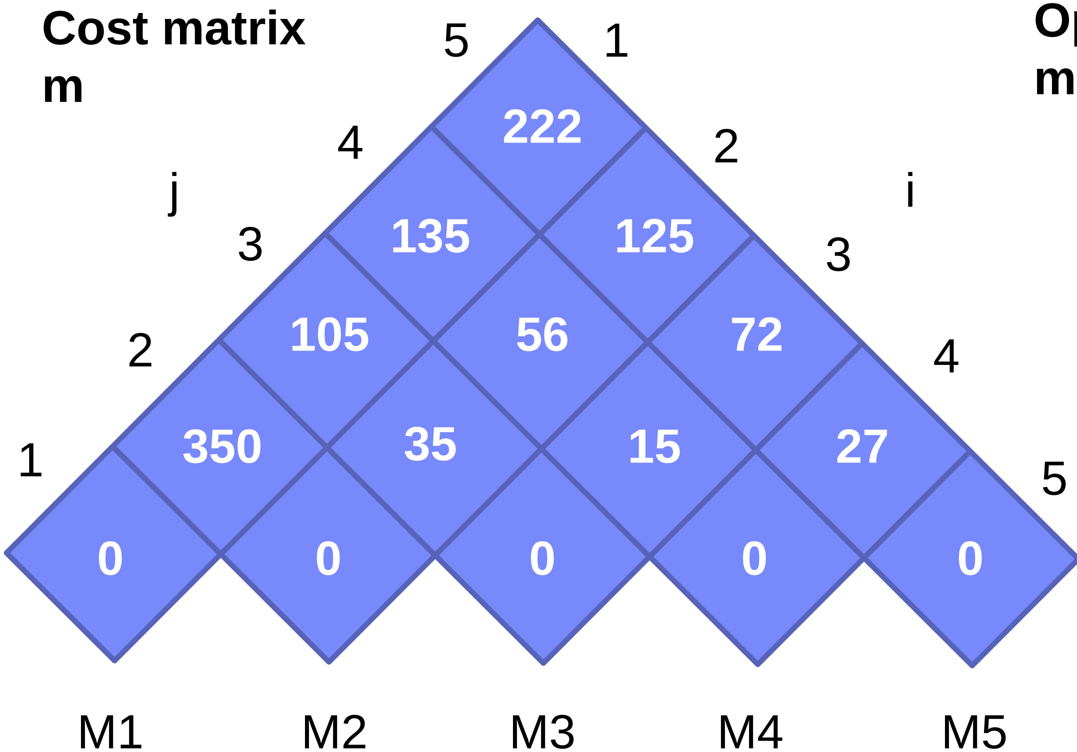
= min(
0 + 35 + 10*7*1,
350 + 0 + 10*5*1)

= min(
105,
400)

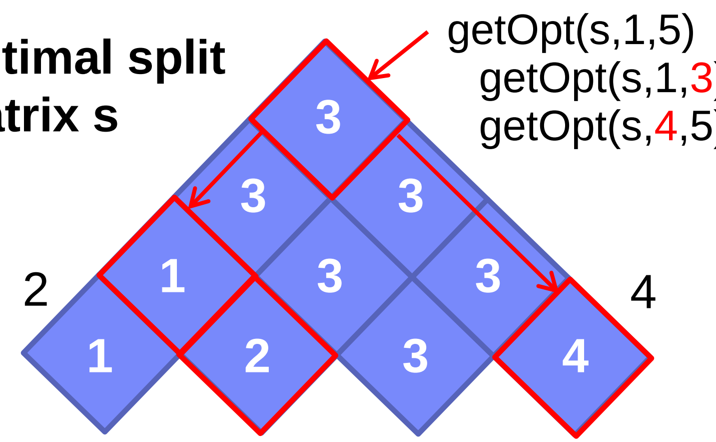
Matrix Multiplication Chain Optimization (3)

M1	M2	M3	M4	M5
10x7	7x5	5x1	1x3	3x9

Cost matrix m



Optimal split matrix s



(M1 M2 M3 M4 M5)

((M1 M2 M3) (M4 M5))

((M1 (M2 M3)) (M4 M5))

➔ ((M1 (M2 M3)) (M4 M5))

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    Y = rand(rows=nrow(X), cols=1, min=1, max=maxval);
    X = cbind(X, table(seq(1,nrow(Y)),Y));}
print(sum(X));
```

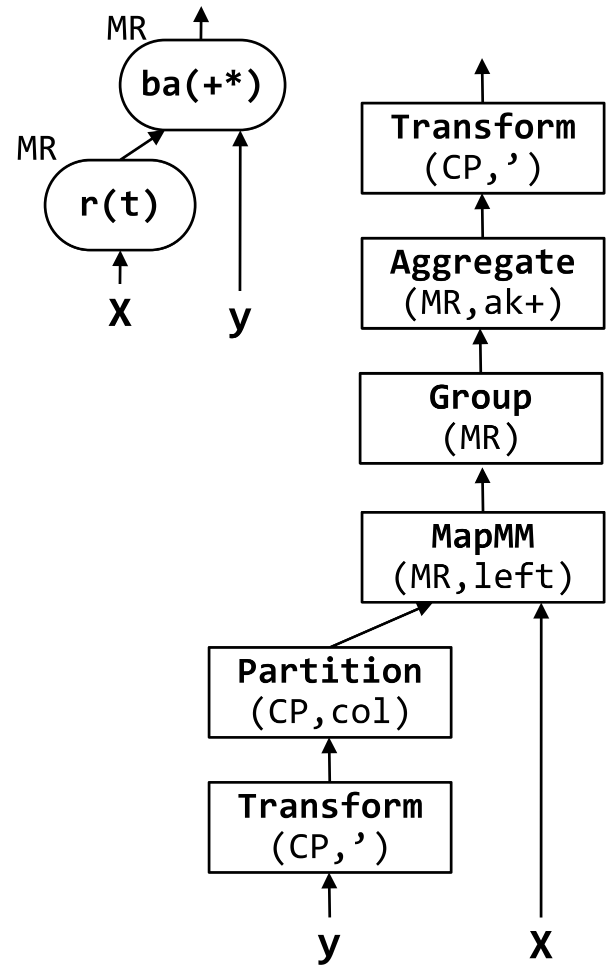
Example Operator Selection: Matrix Multiplication

Exec Type	MM Ops	Pattern

CP	**MM**
	MMChain
	TSMM
	PMM
		`X %*% Y`
		`t(X) (w * (X %*% v))`
		`t(X) %*% X`
		`rmr(diag(v)) %*% X`

MR / Spark
 	
(* only Spark)
	MapMM
	MapMMChain
	TSMM
	ZipMM *
	CPMM
	RMM
	PMM
		`X %*% Y`
		`t(X) (w * (X %*% v))`
		`t(X) %*% X`
		`t(X) %*% Y`
		`rmr(diag(v)) %*% X`
		`X %*% Y`
		`X %*% Y`

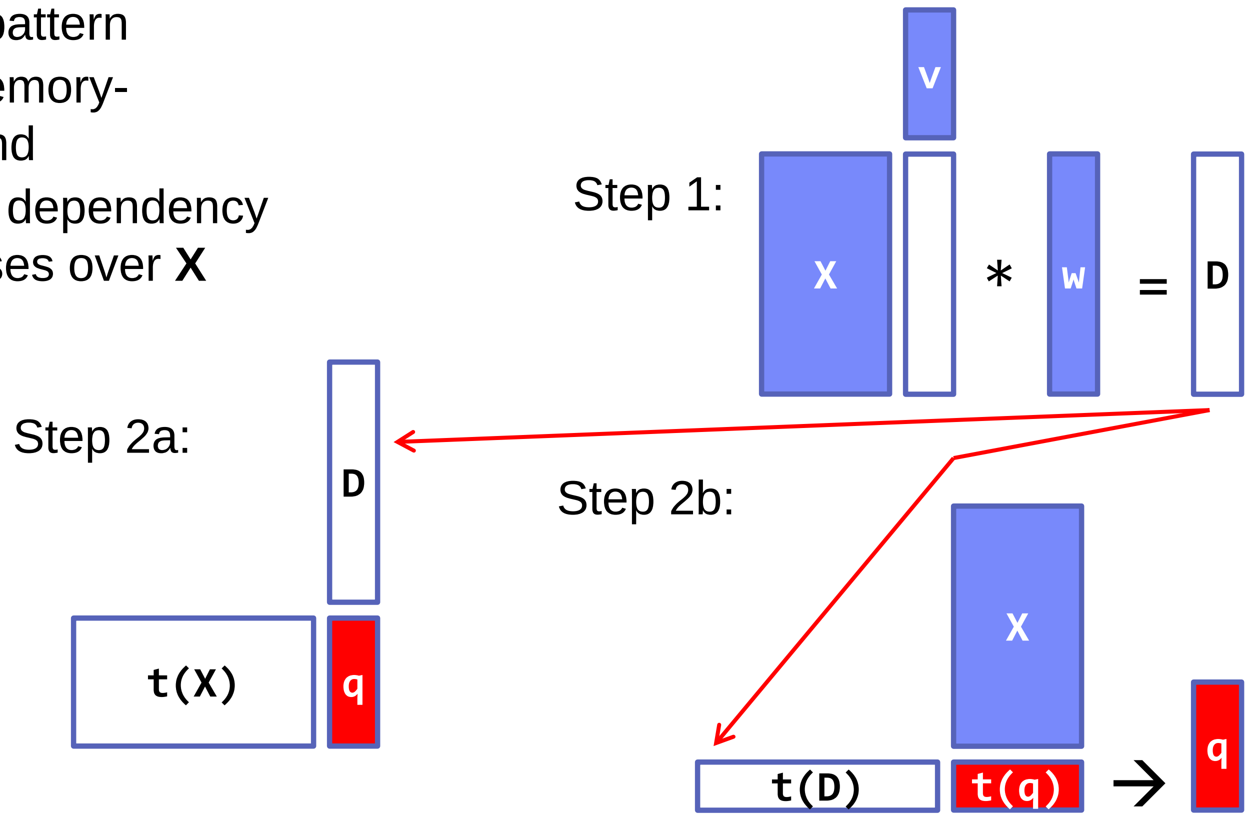
Example: `t(X)%*%y`



- **Hop-Lop Rewrites**
  - Aggregation (w/o, singleblock/multiblock)
  - Partitioning (w/o, CP/MR, col/rowblock)
  - Empty block materialization in output
  - Transpose-MM rewrite t(X)%*%y → t(t(y)%*%X)
  - CP degree of parallelism (multi-threaded mm)

Example Fused Operators (1): MMChain

- **Matrix Multiplication Chains:** `q = t(X) %*% (w * (X %*% v))`
  - Very common pattern
  - MV-ops IO / memory-bandwidth bound
  - **Problem:** Data dependency forces two passes over **X**



[Arash Ashari et al.: On optimizing machine learning workloads via kernel fusion. **PPOPP 2015**]

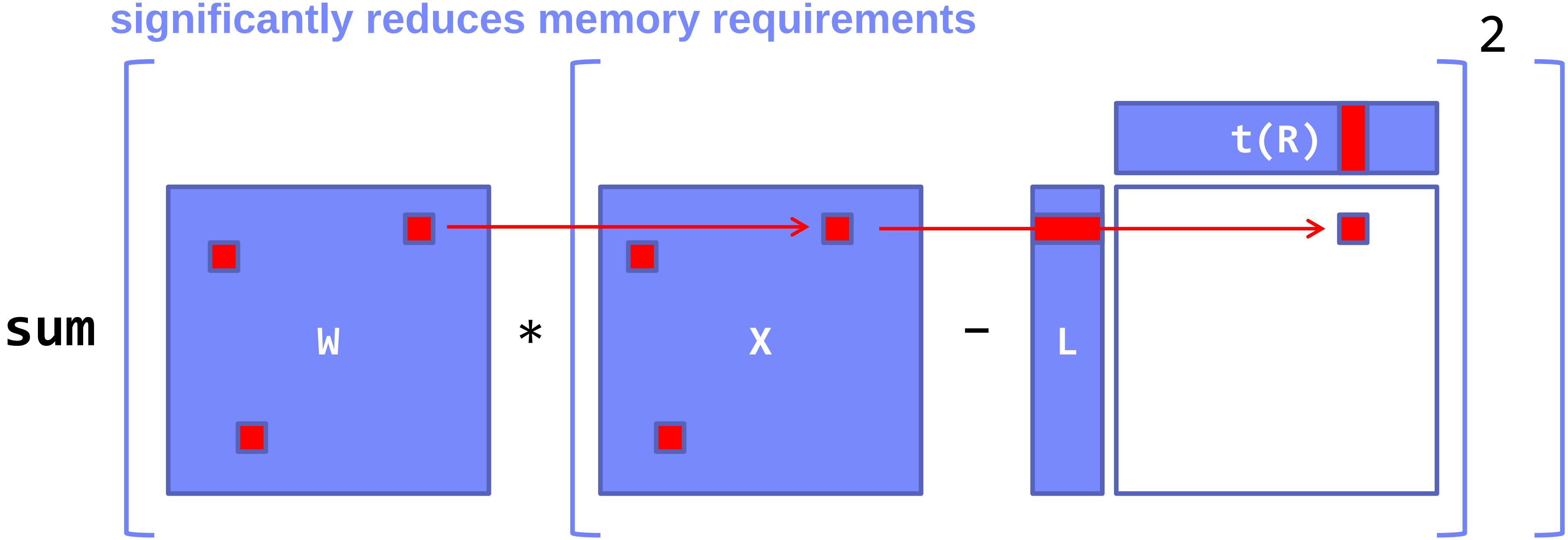
➔ **Fused mmchain operator**
- **Key observation: values of D are row-aligned wrt to X**
- **Single-pass operation** (map-side in MR/Spark / cache-conscious in CP/GPU)

Example Fused Operators (2): WSLoss

- **Weighted Squared Loss:** `wsl = sum(W * (X - L %*% t(R))^2)`
  - Common pattern for factorization algorithms
  - **W** and **X** usually very sparse (< 0.001)
  - **Problem**: "Outer" product of L%*%t(R) creates **three dense** intermediates in the size of **X**

[Matthias Boehm et al.: SystemML: Declarative Machine Learning on Spark. **VLDB 2016**]

➔ **Fused wsloss operator**
  - **Key observations: Sparse W* allows selective computation, full aggregate significantly reduces memory requirements**



Rewrites and Operator Selection in Action

- **Example:** Use case Mlogreg, X: $10^8 \times 10^3$, K=1 (2 classes), 2GB mem
- **Applied Rewrites**
  – Original DML snippet of inner loop:
```
Q = P[, 1:K] * (X %*% ssX_V);
HV = t(X) %*% (Q - P[, 1:K] * (rowSums(Q) %*% matrix(1, rows=1, cols=K)));
```
  – After remove unnecessary (1) matrix multiply (2) unary aggregate
```
Q = P[, 1:K] * (X %*% ssX_V);
HV = t(X) %*% (Q - P[, 1:K] * Q);
```
  – After simplify distributive binary operation
```
Q = P[, 1:K] * (X %*% ssX_V);
HV = t(X) %*% ((1 - P[, 1:K]) * Q);
```
  – After simplify bushy binary operation
```
HV = t(X) %*% (((1 - P[, 1:K]) * P[, 1:K]) * (X %*% ssX_V));
```
  – After fuse binary dag to unary operation (sample proportion)
```
HV = t(X) %*% (sprop(P[, 1:K] * (X %*% ssX_V));
```

Recall: Cascading rewrite effect

- **Operator Selection**
  – Exec Type: **MR**, because mem estimate > 800GB
  – MM Type: **MapMMChain**, because XtwXv and `w=sprop(P[,1:K])` < 2GB
  – CP partitioning of **w** into 32MB chunks of rowblocks

Dynamic Recompilation – Motivation

- **Problem of unknown/changing sizes**
  - **Unknown or changing** sizes and sparsity of intermediates (across loop iterations / conditional control flow).
  - These unknowns lead to very **conservative fallback plans**.
- **Example ML Program Scenarios**
  - Scripts w/ complex function call patterns
  - Scripts w/ UDFs
  - Data-dependent operators

```
Y = table( seq(1,nrow(X)), y )
grad = t(X) %*% (P - Y);
```

  - Computed size expressions
  - Changing dimensions or sparsity

Ex: Stepwise LinregDS

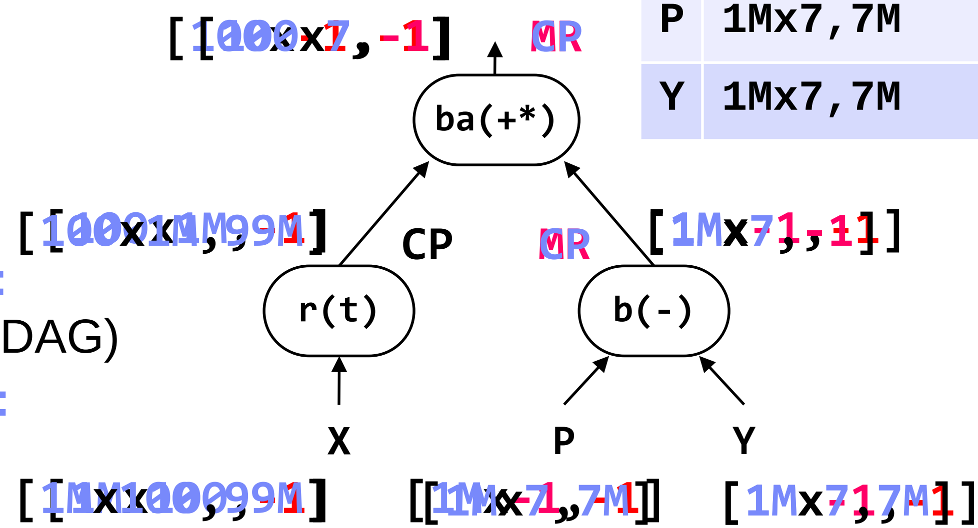
```
while( continue ) {
   parfor( i in 1:n ) {
      if( fixed[1,i]==0 ) {
         X = cbind(Xg, Xorig[,i])
         AIC[1,i] = linregDS(X,y)
      }
   }
   #select & append best to Xg
}
```

➔ **Dynamic recompilation techniques as robust fallback strategy**
  - **Shares goals and challenges with adaptive query processing**
  - **However, ML domain-specific techniques and rewrites**

Dynamic Recompilation – Compiler and Runtime

- **Optimizer Recompilation Decisions**
  - **Split HOP DAGs for recompilation:** prevent unknowns but keep DAGs as large as possible; we split after reads w/ unknown sizes and specific operators
  - **Mark HOP DAGs for recompilation:** MR due to unknown sizes / sparsity
- **Dynamic Recompilation at Runtime** on recompilation hooks (last level program blocks, predicates, recompile once functions, specific MR jobs)
  - **Deep Copy DAG:** (e.g., for non-reversible dynamic rewrites)
  - **Update DAG Statistics:** (based on exact symbol table meta data)
  - **Dynamic Rewrites:** (exact stats allow very aggressive rewrites)
  - **Recompute Memory Estimates:** (w/ unconditional scope of single DAG)
  - **Generate Runtime Instructions:** (construct LOPs / instructions)

X	1Mx100,99M
P	1Mx7,7M
Y	1Mx7,7M



Inter-Procedural Analysis – Motivation

- **Challenges**
  - Multiple function calls with different inputs
  - Conditional control flow
  - Complex function call graphs (incl recursion)

➡ **Inter- and Intra-Procedural Analysis**

- **Example (multiple calls w/ different inputs)**

```
X = read($X1)
X = foo(X);
if( $X2 != " " ) {
    X2 = read($X2);
    X2 = foo(X2);
    X = cbind(X, X2);
} ...
```

1M x 1k

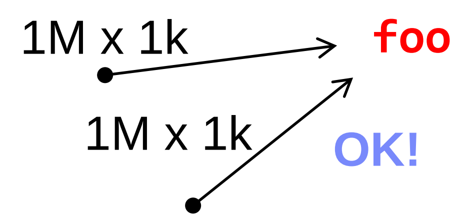
1M x 2

```
foo = function (Matrix[Double] A)
    return (Matrix[Double] B)
{
    B = A - colSums(A);
    if( sum(B!=B)>0 )
        print("NaNs encountered.");
}
```

Size propagation into foo() would be incorrect!

Inter-Procedural Analysis (2)

- **Collect IPA Function Candidates**
  - Functions called once
  - Functions called with consistent sizes (dims/nnz)
  - Unary size-preserving functions



- **Size Propagation (via dynamic recompilation)**
  - Inter- and intra-procedural size propagation (in execution order)
  - Control-flow-aware propagation and reconciliation

- **Additional IPA Passes**
  - Remove unused functions
  - Flag functions "recompile once"

```
foo = function (Matrix[Double] A)
      return (Matrix[Double] C)
{     recompile once on entry w/ A
      B = rand(nrow(A),1);
      while(...)
          C = A / rowSums(A) * B
}
```

  - Remove constant binary operations

```
A = matrix(1, nrow(X), ncol(X));
while(...)
    ... * A
```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    X = cbind(X, table(seq(1,nrow(Y)),Y));}
print(sum(X));
```

From SystemR to SystemML – A Comparison

- **Similarities**
  - **Declarative specification** (fixed semantics): SQL vs DML
  - **Simplification rewrites** (Starburst QGM rewrites vs static/dynamic rewrites)
  - **Operator selection** (physical operators for join vs matrix multiply)
  - **Operator reordering** (join enumeration vs matrix multiplication chain opt)
  - **Adaptive query processing** (progressive reop vs dynamic recompile)
  - **Physical layout** (NSM/DSM/PAX page layouts vs dense/sparse block formats)
  - **Buffer pool** (pull-based page cache vs anti-caching of in-memory variables)
  - **Advanced optimizations** (source code gen, compression, GPUs, etc)
  - **Cost model / stats** (est. time for IO/compute/latency; histograms vs dims/nnz)
- **Differences**
  - **Algebra** (relational algebra vs linear algebra)
  - **Programs** (query trees vs DAGs, conditional control flow, often iterative)
  - **Optimizations** (algebra-specific semantics, rewrites, and constraints)
  - **Scale** (10s-100s vs 10s-10,000s of operators)
  - **Data preparation** (ETL vs feature engineering)
  - **Physical design, transactions processing, multi-tenancy**, etc

SystemML is Open Source:

Apache Incubator Project since 11/2015

Website: http://systemml.apache.org/

Sources: https://github.com/apache/incubator-systemml